

## Problem A.7

Evaluate  $\int \mathbf{r} \cdot (\mathbf{a} - \mathbf{r}) \delta^3(\mathbf{r} - \mathbf{b}) d^3r$ , if  $\mathbf{a} = (1, 2, 3)$ ,  $\mathbf{b} = (3, 2, 1)$ , and the integration is over a sphere of radius 1.5 centered at  $(2, 2, 2)$ .

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### Solution

The aim is to determine whether the point with position vector  $\mathbf{b} = (3, 2, 1)$  lies inside the sphere centered at  $(2, 2, 2)$  with radius 1.5. Calculate the distance between these two points.

$$d = \sqrt{(3-2)^2 + (2-2)^2 + (1-2)^2} = \sqrt{2} \approx 1.414$$

Since it's less than 1.5, the point with position vector  $\mathbf{b}$  lies inside the sphere, and

$$\begin{aligned} \int \mathbf{r} \cdot (\mathbf{a} - \mathbf{r}) \delta^3(\mathbf{r} - \mathbf{b}) d^3r &= [\mathbf{r} \cdot (\mathbf{a} - \mathbf{r})] \Big|_{\mathbf{r}=\mathbf{b}} \\ &= \mathbf{b} \cdot (\mathbf{a} - \mathbf{b}) \\ &= (3, 2, 1) \cdot (-2, 0, 2) \\ &= 3(-2) + 2(0) + 1(2) \\ &= -4. \end{aligned}$$